How a simple equation delivers breakthrough understanding of grinding circuits

by Robert McIvor

The following is a summary of the Robert H. Richards Lecture on "Learning and Good Fortune on the Road to Discovery" presented at the Mineral & Metallurgical Processing Division (MPD) Awards Plenary Session on Feb. 23, 2026 during the MINEXCHANGE 2026 SME Annual Conference & Expo in Salt Lake City, UT.

The ball milling circuit

A breakthrough in new understanding of ball milling circuit pump-mill-cyclones interactions (Fig. 1) has been brought forward by discoveries of "classification system efficiency" and the "functional performance equation."

The pump

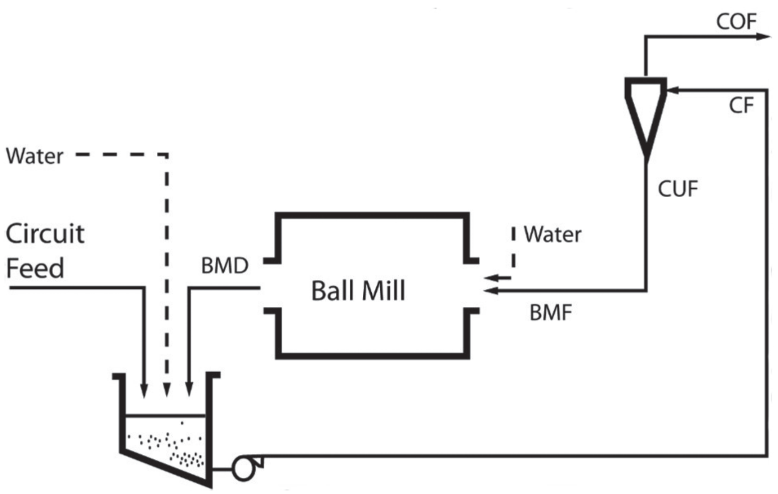
The role of the pump is to deliver slurry to the cyclones at a specified flow rate and pressure. Upon graduation from McGill University, McIvor was hired into the pump department at Allis-Chalmers Canada Ltd. New to the field of slurry pump process application engineering, McIvor was trained how to calculate pumping system total dynamic head (TDH) using the Bernoulli equation, and to select the pump size, speed and motor required to deliver the specified flow rate. Whether a grinding circuit, or another processing plant application, slurry pump process design methods became ingrained, and would prove essential for the discoveries to come.

The mill

The role of the ball mill is to deliver energy that results in feed material breakage. It does so indiscriminately, a cylinder loaded with grinding balls rotated by its drive motor from which it demands power, irrespective of what it is fed. New to the field of grinding mill process application engineering, McIvor was trained how to calculate the specific energy (kWh/t) needed to reduce ore in a ball milling circuit from a given feed (80 percent passing or F80) size to a desired (80 percent passing or P80) product size using the Bond work index equation (Bond, 1952):

Figure 1

The ball milling circuit.



$$W = WI \times \left(\frac{10}{\sqrt{P80}} - \frac{10}{\sqrt{F80}}\right)$$

where W is the needed work (energy) per ton, and WI is the laboratory work index.

Fred C. Bond directed process application engineering at Allis-Chalmers at the time, retained as a consultant after his retirement, at headquarters in Milwaukee. The mill application engineer's job is to select a ball mill that will demand the amount of power that is equal to that needed to grind the ore as determined from the Bond's work index equation. It was based on his correlation between plant operating work indices and his laboratory test work indices, in addition to a table of ball mill power based on dimensions, speed and ball loading.

The grinding circuit was considered a "black box," containing the mill needed to draw the specified power, and a pump and the classification equipment to operate in closed circuit. It was recognized that the method

Editor's note: Robert McIvor is the recipient of the 2026 Robert H. Richards Award, which recognizes achievement that unmistakably furthers the art of mineral beneficiation. He earned degrees in mining engineering at the University of Saskatchewan and McGill University, and a doctorate from McGill University for studies in mineral grinding. He is a Fellow of AIME, has received the Art MacPherson Medal from CIMM and is co-recipient of the CEEC Writing Medal for Operations Improvement.

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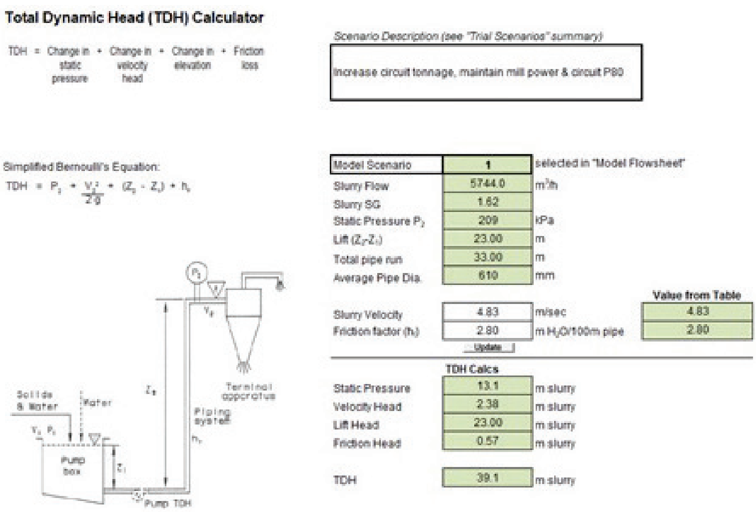
Figure 2

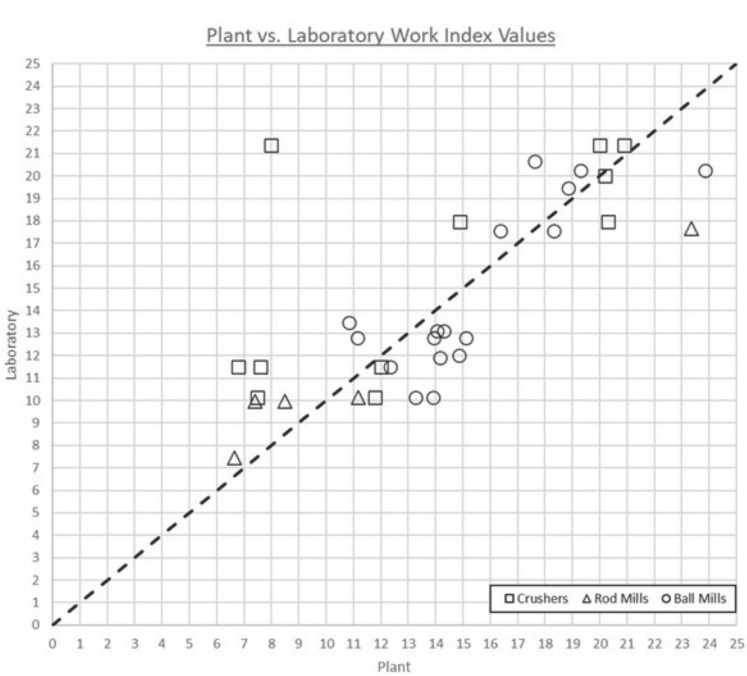
Example slurry pumping system flow and TDH.



depended on circuit feed and product size distributions that were close to similar in shape when plotted on a log-log scale, a fact broadly observed in plant data. The job was to select the mill, and no consideration was given to the pump or cyclones. Although there is an inherent assumption of overall plant circuit efficiency based on Bond's regression line equating plant operating work index to the ore's laboratory work index, the success of the method is well established. Much more often than not, the mill would readily do the job. Design safety factors were built into new projects, and scale-up inaccuracies become insignificant compared to variations in feed ore characteristics. Plant control systems would maximize tonnage while targeting the desired grind, contributing to efficient performance of plant installations.

Figure 3

Bond's plant WI and laboratory test WI correlation (1952).



The cyclones

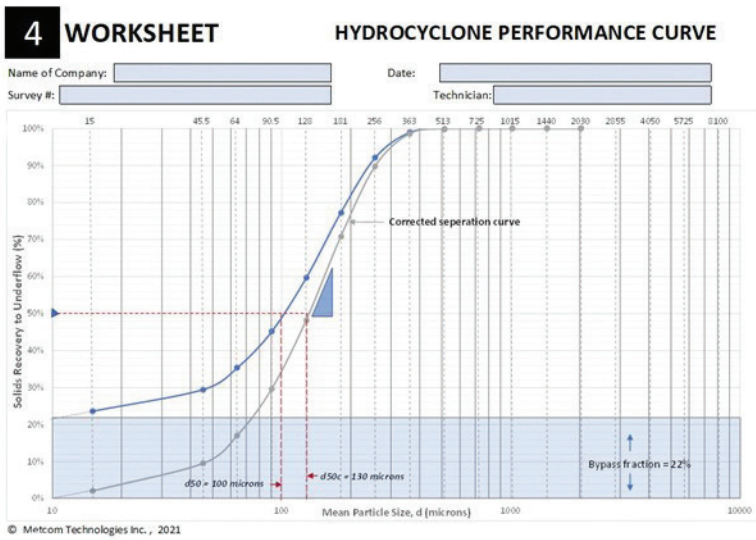
The role of the cyclones is to remove product size material from the circuit while returning oversize to the mill for further grinding. After a decade in grinding mill application design, McIvor accepted the position of head of the slurry pump and cyclone equipment division at Linatex Canada Ltd. Slurry pumping knowledge gained at Allis-Chalmers was immediately useful. Linatex engineers in South Africa and Australia were early developers of hydrocyclone technology, and leading contributors to their adoption by the minerals industry. The opportunity to learn about cyclones was a key attraction of this new job.

To say that gaining mastery over hydrocyclone process design was a challenge is a gross understatement. But with persistence, the knowledge generously shared by the experts at Linatex and a critical work by Plitt (1971), a methodology for grinding circuit cyclone selection was pieced together (McIvor, 1984) and is briefly summarized as follows:

1. The cyclone overflow solids rate is the grinding circuit tonnage. The underflow solids rate is the overflow solids rate multiplied by the specified circulating load ratio. The feed solids rate is the sum of the two.
2. The cyclone overflow water rate (percent solids) is defined by the downstream requirement. The underflow should be fifty percent solids by volume. The feed water rate is the sum of the two.
3. The cyclone overflow, underflow and feed size distributions also can be estimated. The overflow size distribution passes through the target 80 percent passing size (P80) and takes on the usual log-log plotted shape. The cyclone feed size distribution also follows a typical shape. So does a typical cyclone separation performance curve (Fig. 4), with bypass dictated by the water balance. By trial and error of both, find the cyclone feed size distribution and cyclone separation performance that splits the feed solids into overflow (at the target P80) and underflow at the specified circulating load ratio.
4. The estimated cyclone size distributions

Figure 4

Example cyclone size separation performance.



match the cyclone separation performance, including the d50c, the size that reports 50 percent to each product stream (without the bypass). The Plitt (1971) equation relates d50c to cyclone dimensions and feed density and flow rate. Select cyclone dimensions that provide the d50c, and select the number of cyclones that provides the total flow rate at a suitable feed pressure.

Changing the circulating load ratio

With the above tools in hand, McIvor was capable of modifying (or changing out) the cyclones and pump in a grinding circuit to achieve a newly specified circulating load ratio. Davis (1925) had shown that high circulating load greatly contributes to increased grinding circuit productivity. Identifying those plants with low circulating load created the opportunity for improved grinding circuit performance in the plants, and new equipment sales for Linatex.

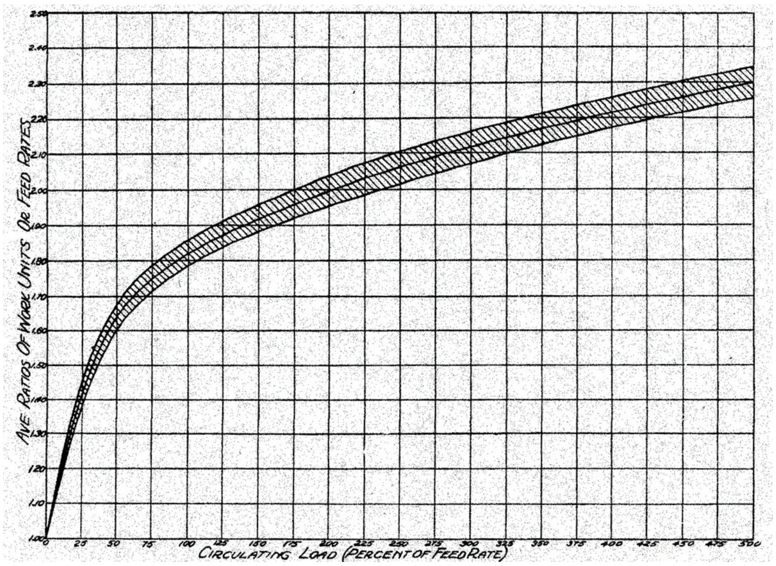
Discovery of circuit classification system efficiency (CSE)

It was while plotting mill feed and discharge size distributions from two operations with distinctly different circulating load ratios that McIvor experienced a “Eureka!” moment.

Given the same target P80 of 106 µm for both circuits, the ball mill at low circulating load contained approximately 75 percent plus

Figure 5

The Davis circulating load — circuit productivity relationship circuit (1925).



106 µm, while the ball mill at high circulating load contained approximately 85 percent plus 106 µm. It follows that the portion of the mill energy being put to good use rises from 75 to 85 percent with the higher circulating load ratio. Akin to mill motor drive efficiency, ball milling circuit "classification system efficiency" (CSE) could be defined as the percentage of "coarse" material (larger than the circuit P80 size) within the mill. Davis (1945) had also shown that the size distribution of the mill contents was well represented by the average of the mill feed and discharge. CSE could be measured readily, and then predictably increased through suitable pump and cyclone adjustments. Cyclone water usage and physical design features affecting sharpness of separation have since been added to circulating load ratio as key grinding circuit CSE improvement opportunities (McIvor et al., 2017).

Discovery of the functional performance equation

McIvor brought the concept of CSE to McGill University for further study and development. Following the work of Miles (1972) in a class on "Value Analysis and Engineering," discussions with teaching staff and fellow students led to the idea of defining the function of the pump and cyclones to provide high CSE in the ball mill. What, then, would be the function of the ball mill?

Distinguishing between "fines" (product size material) and "coarse" material needing further size reduction, it was first simply stated that production of new fines was the function of the ball mill. These fines are generated by application of mill power onto coarse material: production rate of fines (t/h) = power applied to coarse (kW) × mill grinding rate of coarse (t/kWh). However, the power applied to the coarse material in the mill is the total mill power multiplied by the CSE. Therefore: production rate of fines (t/h) = mill power (kW) × classification system efficiency (percent) × mill grinding rate of coarse (t/kWh).

It was noted that by dividing the mill grinding rate by the grindability of the ore, such as the grams of new product per revolution measured in a Bond work index test, this ratio provides a measure of the grinding efficiency of the ball mill. To balance the equation, also multiply by the ore grindability. Therefore: production rate of fines (t/h) = mill power (kW) × classification system efficiency (percent) × ore grindability (g/rev) × mill grinding efficiency of coarse (t/kWh)/(g/rev).

This was the birth of the "functional performance equation" for ball milling circuits. It identifies and decouples two, separate efficiencies that are in play in the grinding circuit. Combined with the mill power and the grindability of the ore, total production rate of the circuit is defined. Importantly, first, the equation is readily completed by carrying out a plant circuit survey. Secondly, each efficiency is clearly related to specific design and operating variables: CSE, by the setup of the pump and cyclones, and mill grinding efficiency by conditions within the mill (such as percent solids, media sizing and liner design). Each efficiency can be measured, related to design and operating variables, which can then be manipulated, and each efficiency remeasured in a sequence of optimization steps.

Putting it to work

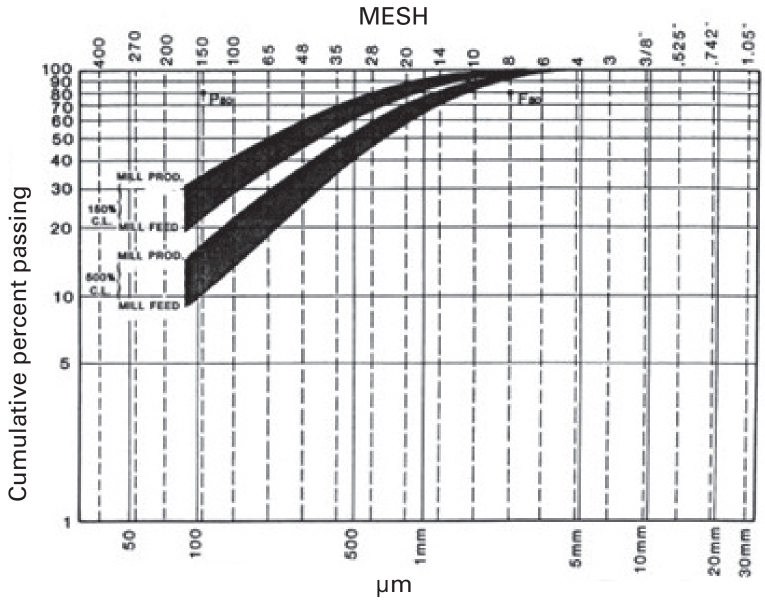
Achievement of plant processing performance improvements through application of the functional performance equation requires completion of a list of related tasks that must be carried out with a high level of diligence. McIvor turned to the use of training to deliver the needed skill set, broadly, to those working with plant grinding circuits. The first major step in the development of a metallurgist training program was a formal "task analysis." Working with instructional design specialists, and partnering with two mineral processing plant operations and a media supplier, 21 distinct skills were identified that are needed to carry out a systematic plant grinding circuit processing performance improvement project. Examples are characterizing mill power draw; plant sampling methods and conducting plant circuit surveys; sample analyses, including grindability testing; work index efficiency and functional performance equation calculations; and adjusting pump and cyclone performances, to list a few. A complete list may be found in the paper by Lavalee et al. (1992). This training program has been certified by The Engineering Institute of Canada, continually upgraded (Arafat et al., 2015), and to date has been provided to more than 1,000 plant metallurgists, equipment suppliers and plant designers worldwide. It is available in three languages, online, through Metcom Technologies Inc.

Summary and conclusion

The utility of the functional performance equation for making plant grinding circuit improvements comes about as a result of certain key characteristics. It is simple. It is accurate, each element a linear input to circuit production rate. It clarifies understanding of grinding circuits, showing that there are actually two separate efficiencies in play. It provides measurements for each. Each efficiency is clearly linked to specific circuit design and operating variables which can be manipulated to increase

Figure 6

Mill feed and discharge size distributions at low and high circulating load.



them. Maximizing each efficiency are clear goals. A strategy for plant grinding processing performance improvement is provided.

Application of the functional performance equation for improving plant grinding performance is becoming widespread. (For more details, see *Mining Engineering* articles "Ball Mill Classification System Optimization Through Functional Performance Modeling" November 2017, and "Ball mill media optimization through functional performance modeling" November 2018.) ■

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References

Bond, F.C., 1952, "The Third Theory of Comminution", AIME Trans, v. 193, p.484-494.

Plitt, L. R., 1971, "The Analysis of Solid-Solid Separations in Classifiers", *CIM Bulletin*, April, 1971, p.42-47.

McIvor, R.E., 1984, "A Material Balance Calculation Procedure for Grinding Circuit Hydrocyclone Selection", CIM Bulletin, December, p. 50-53.

Davis, E.W., 1925, "Ball Mill Crushing in Closed Circuit with Screens", Bulletin of the University of Minnesota, School of Mines Experimental Station", v. 28, no. 42, p. 14-18.

Davis, E.W., 1945, "Pulp Densities within Operating Ball Mills", AIME Tech. Pub. No. 1843.

Miles, L.D., 1972, Techniques of Value Analysis and Engineering, second edition, McGraw Hill, New York.

Lavalee, M. L., McIvor, R.E., and Stolovitch, H. D., 1992, "The Design of Instruction to Introduce a New Technology", Proceedings of the Canadian Mineral Processors Annual Conference, Ottawa.

Arafat, O.A., Bartholomew, K.M., and McIvor, R.E., 2015, "Content and Structure of a Plant Grinding Performance Improvement System", Proceedings of the Canadian Mineral Processors Annual Conference, Ottawa.

McIvor, R.E., Bartholomew, K.M., Arafat, O.A., and Finch, J.A., 2017, "Ball Mill System Classification Optimization Through Functional Performance Modelling", *Mining Engineering*, Nov, p. 18-31.

Conger, W., Dupont, J-F., McIvor, R.E., and Weldum, T.P., 2018, "Ball Mill Media Optimization Through Functional Performance Modeling", *Mining Engineering*, Nov, p. 28-38.